2008:

1. A standard warm-up question about vectors. The ball has a downwards velocity due to the throw, and a rightwards velocity from the car. Thus the answer is B

2. Kinematics: $s = ?$ $u = 10$ $v =$ $a = g (\simeq 10 m s^{-2})$ $t = 2$
Watch out; the ball is thrown horizontally with a speed of $20 m s^{-1}$, but that has no bearing on the vertical speed (which is zero). Use $s = ut + \frac{1}{2}at^2$ to get 19.6m. Thus the answer is D.

3. Remember that for power dissipation $P = IV$. Use Ohm's Law ($V = IR$) to derive $P = \frac{V^2}{R}$, noting that R is constant as we're using the same resistor.
Thus the answer is E.

4. Using your right-hand, point your thumb in the direction of the current. Your fingers curl to give the magnetic field lines that form circles around the wire. This acts perfectly along the loop, giving no force.
Thus the answer is E.

5. De Broglie's formula is $\lambda = \frac{h}{p}$, where h is Planck's constant.
If stuck, first reason it out with dimensional analysis.
λ has units m $([M]^0[L]^1[T]^0)$, p has units $kg\, m s^{-1}$ (from $p = mv$) $([M]^1[L]^1[T]^{-1})$.
Then if $\lambda = Ap$, A has units $kg^{-1} s$ (or rewriting this as $A\lambda = p$, A has units $kg\, s^{-1}$).
Alternatively, we could have $p\lambda = A$, where A has units $kg\, m^2 s^{-1}$.
Running through the options given, h has units $kg\, m^2 s^{-1}$, and none of the other options work with any of the cases.
Thus the answer is A.

6. Thinking like a chemist, remember that $n=1$ has 2 electrons (thinking of hydrogen) and $n=2$ has 8 electrons (e.g: thinking of carbon).
More formally, we write electronic structure as
$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} \ldots$ and so on, where the large numbers correspond to n, the letters correspond to l (irrelevant here), and the superscripts give the specific electron.
$1s^2 2s^2 2p^6$ is relevant to $n=1$ and $n=2$, and $2+2+6 = 10$.
Thus the answer is E.

7. ~~It is a standard fact that $V_{rms} = \sqrt{\frac{3kT}{m}}$ in an ideal gas~~
~~At the very least, you should realise that zero isn't the answer (remember it's speed, not velocity that the question is asking for), and you might guess the factor of $\sqrt{3}$ comes from having 3 dimensions,~~
~~Thus the answer is C.~~

8. At first glance, discussion of cavities and Kelvin temperatures may make this question seem unfamiliar. Don't stop thinking though, it's just a Stefan-Boltzmann question! Recall that $P = A\sigma T^4$, so if T doubles P increases by a factor of 16, and this intensity correlates with melting.

Thus the answer is D.

9. The diagram should remind you of Kepler's 2nd Law. In fact: I = Kepler's 2nd Law; II = Kepler's 1st Law; III = Kepler's 3rd Law. Thus the answer is E.

10. Think of energies! $E_K = \frac{1}{2}mv^2$, $E_e = \frac{1}{2}ks^2$. Therefore $\frac{1}{2}mv^2 = \frac{1}{2}ks^2 \Rightarrow s = \sqrt{\frac{mv^2}{k}} = v\sqrt{\frac{m}{k}}$. If you're stuck, note that (A), (D) and (E) don't hold under dimensional analysis. Thus the answer is (B).

11. ~~This is a standard result~~: $E_n = (n+\frac{1}{2})\hbar\omega$, where $n=0$ ~~gives the ground~~. Therefore $E_{ground} = \frac{1}{2}\hbar\omega$

~~Thus the answer is C.~~

12. Recall that angular momentum is quantised in the Bohr model, such that ~~$mvr = n\hbar$~~ $L = n\hbar$. Also $L = mvr_n$ so $mvr_n = n\hbar$. We want linear momentum $p = mv$ though, so rearrange: $mv = \frac{n\hbar}{r_n}$. Thus the answer is C.

13. Just read two points on the graph and approximate: (3,10) and (10000, 100) are the end points. This gives $\approx y = \frac{1}{2}x + 6$. A quicker way in this case is to extrapolate the line backwards to the y-axis, and note the intercept is $\approx y = 6$. Then, for a line of the form $y = mx + c$, $c = 6$.

Thus the answer is B.

14. The formula for the weighted average is $\frac{\frac{\overline{x_A}}{\sigma_A^2} + \frac{\overline{x_B}}{\sigma_B^2}}{\left(\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2}\right)}$ where the notation is $\overline{x} \pm \sigma$ (i.e: $\overline{x}_A = 11$ kg, $\sigma_A = 1$ kg, etc). Plugging this in, we get $\frac{\left(\frac{11}{1^2} + \frac{10}{2^2}\right)}{\left(\frac{1}{1^2} + \frac{1}{2^2}\right)} = \frac{4}{5}(11 + 2.5)$

The more general formula is $\overline{x} = \frac{\sum_{i=1}^{n}(x_i \sigma_i^{-2})}{\sum_{i=1}^{n} \sigma_i^{-2}}$

$= \frac{54}{5}$ The uncertainty is calculated by $\sigma_x = \sqrt{\frac{1}{\sum \sigma_i^{-2}}}$

$\Rightarrow \sigma_x = \sqrt{\frac{1}{\frac{1}{1^2} + \frac{1}{2^2}}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$ kg. Thus the answer is B.

15. By the lensmaker's eqn: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. WLOG, take $n=2$.

For (A): R_1 (left) ≥ 0 $R_2 < 0$
(B): $R_1 = \infty$ $R_2 < 0$
(C): $R_1 = \infty$ $R_2 > 0$
(D): $R_1 \leq 0$ $R_2 > 0$
(E): $R_1 < 0$ $R_2 > 0$

The answer is between (D) and (E). Clearly $R_{1D} > R_{1E}$, so The answer is E.

16. When unpolarised light is incident on a filter, the intensity drops by half. Next, the drop in intensity by the 2nd filter is given by Malus' Law: $I = I_0 \cos^2(\theta)$, where $\theta = 45 \Rightarrow I = \frac{1}{2} I_0$. So it halves again, giving 25% of the initial intensity.
Thus the answer is D.

17. Using Gauss's Law: $\int \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} \Rightarrow |\vec{E}| 2\pi r l = \frac{\lambda l}{\varepsilon_0}$
$\Rightarrow |\vec{E}| = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r}$
Thus the answer is A.

18. This is a standard question assessing understanding of Lenz's Law. As the magnet passes through the loop to halfway, the flux through the loop increases, so current moves to oppose the flux (i.e. B to A) then it reverses.
Thus the answer is E.

19. Rather nicely, this is a question about Wien's Law that doesn't actually require you to know Wien's Law. Typically, memorise $\lambda_{max} = \frac{2.9 \times 10^{-3}}{T}$.
Here, if you realise that $E \sim T$ and $E \sim f \sim \frac{1}{\lambda}$ then $\lambda = \frac{A}{T}$, where $A = 6000 \cdot 500 \times 10^{-9}$. (note dimensional analysis isn't your friend here, as Wien's constant is 2.9×10^{-3} mK – without knowing these units you're stuck!). Either way, $\lambda_{max} \sim \frac{3 \times 10^{-3}}{300} = 1 \times 10^{-5}$ m $= 10\mu$m
Thus the answer is A.

20. First of all, even if you don't know any relation, realise that the universe is expanding over time, and it is cooling as it does so. Then (B), (E), (C) make no sense. As a matter of fact, $T \propto a^{-1} \Rightarrow T \propto \frac{1}{1+z} \Rightarrow T = T_0(1+z)$ where T_0 is the current temperature of the universe (which is a fact you should know) $T_0 = 2.7$K. Thus the answer is A.

21. Remember for a polytropic process, $pV^n =$ constant, where if $n = \gamma = c_p/c_v$, the process is adiabatic. Additionally, for an ideal gas, $pV = nRT$. Therefore $\left(\frac{nRT}{V}\right) V^\gamma =$ constant,
$\Rightarrow nRTV^{\gamma-1} =$ constant, and as n and R are constants, $TV^{\gamma-1} =$ constant. Thus the answer is C.

22. The relation is $E^2 = (pc)^2 + (mc^2)^2$. If $E = 4mc^2$, then
$(4mc^2)^2 = (pc)^2 + (mc^2)^2 \Rightarrow 16(mc^2)^2 = (pc)^2 + (mc^2)^2$
$\Rightarrow 15(mc^2)^2 = (pc)^2 \Rightarrow p^2c^2 = 15m^2c^4 \Rightarrow p = \sqrt{15}mc$
Thus the answer is C.

23. This is a standard SR question also designed to test your conceptual knowledge. For Lorentz contraction, $L_0 = \gamma L$ (i.e: As $v \to c$, $L \to 0$). If $L_0 = 1m$ and $L = 0.6m$, ~~γ = 1/0.6~~. $\gamma = \frac{1}{0.6}$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{0.6} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = 0.6 \Rightarrow v = 0.8c$$

Now using the velocity addition formula: $w = \frac{u+v}{1+\frac{uv}{c^2}}$ $(u=v)$ $\Rightarrow$

Solving gets $v = 0.5c$

Thus the answer is B.

24. Again, from $L = \gamma L'$, $v = 0.8c \Rightarrow L = 0.6m$. Then $\Delta t = \frac{L'}{\gamma}\frac{1}{v}$

$\Rightarrow \Delta t = \frac{3}{5 \cdot 0.8c} = 2.5\mu s$

Thus the answer is B

25. For orthogonality: $\int_{-\infty}^{\infty} \psi_i^*(x)\,\psi_j(x) = 0$ where $i \neq j$ and $\int_{-\infty}^{\infty} \psi_i^*(x)\,\psi_i(x) = 1$. Using the kronecker delta, where

$$\delta_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases} \quad \text{then } \int_{-\infty}^{\infty} \psi_i^*(x)\,\psi_j(x) = \delta_{ij}.$$

(Alternatively, we can write this using Dirac notation which may be more familiar from the case of ~~xxx~~ ladder operators. Orthogonality implies $\langle \psi_m | \psi_n \rangle = 0$ if $m \neq n$, $\langle \psi_n | \psi_n \rangle = 1$). This is a relatively common (and easy question), so be familiar with it.

Thus the answer is E.

26. A good piece of knowledge to know is the answer to this question, $r = a_0$. (This comes up relatively often). The derivation is as follows:

$P = \int |\psi(x)|^2 d^3\vec{r} = \int |\psi(x)|^2\, 4\pi r^2 dr \Rightarrow \frac{dP}{dr} = 0 \Rightarrow \frac{d\psi^2}{dr}(4\pi r^2) + 8\pi r|\psi|^2 = 0$. Noting that $\psi_{100} \Rightarrow |\psi|^2 = \frac{1}{\pi a_0^3} e^{-\frac{2r}{a_0}}$ and rearranging, one finds that $r = a_0$.

Thus the answer is D

27. Remember that as a rough approximation: $\Delta E \Delta t \sim \hbar$
$\Rightarrow$ ~~(hν)~~ $\Delta t \sim \hbar \Rightarrow \Delta\nu \sim \frac{1}{\Delta t} = \frac{1}{1.6\times 10^{-9}} \sim 600\,MHz$.
Note that this could be reasonably guessed purely through dimensional analysis

Thus the answer is C

28. $W = F\Delta x$, where $F = -kx$ (Hooke's Law) for the 1st spring, giving $W = -kx^2$. For the second; ~~For the second~~

$$2W = -(nk)\left(\frac{x}{2}\right)^2 \Rightarrow W = -\left(\frac{k}{8}\right)x^2$$

$\Rightarrow 8k$ thus the answer is D.

29. Note that the collision is elastic, meaning kinetic energy is conserved.
Then $\frac{1}{2}Mv^2 = \frac{1}{2}M\left(\frac{v}{2}\right)^2 + \frac{1}{2}Mv_2^2$

$\Rightarrow v_2^2 = v^2\left(1-\frac{1}{4}\right)$

$\Rightarrow v_2 = \frac{\sqrt{3}}{2}v.$

We don't need to set up the momentum equation in this case.
Thus the answer is C.

30. Standard result: In Hamiltonian dynamics,

$$\dot{q} = \frac{\partial H}{\partial p_i}, \quad \dot{p} = -\frac{\partial H}{\partial q_i}.$$

Thus the answer is D.

31. Recall that for fluids: $F = \rho V g$ where ρ is the density of the fluid.
As the block is in equilibrium, we know that:
$\rho_{block} V g = \rho_{H_2O}\left(\frac{3V}{4}\right)g + \rho_{oil}\left(\frac{V}{4}\right)g$, which can be rearranged to get
$\rho_{block} = 950\ kg\,m^{-3}$.
Thus the answer is C.

32. By fluid continuity: $\rho_1 A_1 v_1 \Delta t_1 = \rho_2 A_2 v_2 \Delta t_2$.
Consider Bernoulli's Law: $\frac{1}{2}\rho_1 v_1^2 + \rho_1 g h_1 + P_1 = \frac{1}{2}\rho_2 v_2^2 + \rho_2 g h_2 + P_2$.
Terms in this equation always cancel out on the GRE. We assume that the fluid is incompressible s.t. $\rho_1 = \rho_2$, and the centre of both sections is at $h_1 = h_2$
$\Rightarrow P_2 = P_1 + \frac{\rho v_1^2}{2} - \frac{\rho v_2^2}{2}$. Now use fluid continuity, to derive $P_2 = P_1 - \frac{15}{2}\rho v_1^2$
$\Rightarrow$ Thus the answer is A.

33. The options (if nothing else), should suggest you need to use $\Delta Q = mc\Delta T$.
In addition, by the 2nd law of thermodynamics, $\Delta S \geq \frac{dQ}{T}$

$\Rightarrow \Delta S \geq \int_{T_1}^{T_2} \frac{mc\,dT}{T} = mc\ln\frac{T_2}{T_1}$ Thus the answer is E.

34. Recall that $C_V = \left(\frac{dQ}{dT}\right)_V = \left(\frac{dU}{dT}\right)$ and $C_P \equiv \left(\frac{dQ}{dT}\right)_P = \left(\frac{dU}{dT}\right) + \frac{P\,dV}{dT}$. The gas is monatomic so $U = \frac{3}{2}NkT \Rightarrow C_V = \frac{3}{2}Nk$, $C_P = \frac{3}{2}Nk + P\frac{dV}{dT}$ where $\frac{dV}{dT} = \frac{Nk}{P}$ from the ideal gas law. Therefore $\Delta Q_T = \frac{5}{3}Q$ as $\Delta T = \frac{\Delta Q_P}{C_P} = \frac{\Delta Q_V}{C_V}$.
Thus the answer is C.

35. First, temperatures should be in kelvin, where the conversion is $C + 273 = K$.
Therefore $T_H = 300K$, $T_C = 280K$. Assuming a heat pump of 100% efficiency (so the smallest amount of work is done) then

$$\varepsilon = \left|\frac{W}{Q_H}\right| = \left|\frac{T_H - T_C}{T_H}\right| = 1 - \left|\frac{T_C}{T_H}\right| \Rightarrow W = Q_H\left(1 - \left|\frac{T_C}{T_H}\right|\right)$$

we are $W = 1000J \Rightarrow$ (B)

36. You should intuitively be aware that we are looking for a sinusoidal graph, and the inductor should start with no energy.

Thus the answer is A.

37. Thinking solely about vector direction, E is in the $-\hat{x}$ direction. Additionally you should expect a dependence on ℓ

Thus the answer is E.

38. You could do some force calculations here, but first of all, I would narrow down the options by looking at the direction of the field. Using the right hand rule for magnetic fields, curling your fingers around the ~~box~~ vertical current gives a field line into the page, and curling your fingers for the horizontal current gives an equal field line out of the page. As such, these forces cancel to give zero, and no actual calculations are necessary.

Thus the answer is E.

39. If $v = 0.8c$, then $\gamma = 0.6$ (As a quick aside, this pair of numbers comes up so much that I have it memorised). Now, by time dilation, $t_0 = \gamma t$, where t_0 is the <u>lab frame</u>.

$\cancel{\Rightarrow t_0 = 0.6 \cdot 2.2\times10^{-6}}$ Now, $\Delta x_0 = vt_0$

$\Rightarrow \Delta x_0 = \frac{4c}{5} \cdot 0.6 \cdot 2.2\times10^{-6} \approx \underline{880m}$

Thus the answer is C.

40. I would carefully study this example if you are struggling with the SR. We know $E^2 = (pc)^2 + (mc^2)^2$. Now, looking at the four-momentum vector (written in the form $(\frac{E}{c}, p_x, p_y, p_z)$), ~~$P_{init} = P_{fin}$~~ $P_1 = (\frac{E}{c}, p, 0, 0) = (\sqrt{p^2+(mc)^2}, p, 0, 0)$ $P_0 = (Mc^2, 0, 0, 0)$. From $P_0 = P_1 \Rightarrow Mc^2 = p + \sqrt{p^2+m^2c^4}$. Square both sides to get $M^2c^4 + p^2 - 2pMc^2 = p^2 + m^2c^4$, and rearrange to get $\underline{p = \frac{c^2(M^2-m^2)}{2m}}$.

Thus the answer is B.

41. Recall that for the photoelectric effect, $E = hf - \phi$, i.e: the energy conferred to an electron is the energy of the incident γ minus the energy required to free it from the metal (i.e: the 'work function'). Now $E = qV$ ~~$f=$~~, and rewriting this all in the form $y = mx + c$ where x is the frequency and y is the stopping potential, we have

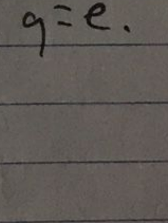
$V = \frac{h}{q} f - \phi/q$. The slope of the line $y = mx + c$ is m, which corresponds to h/q, and then recall that for an electron, $q = e$.

Thus the answer is B.

42. $2 \cdot \frac{2\pi}{6} = \frac{2\pi}{3}$ rad $= 120°$.

Thus the answer is E.

43. You should know from a solid-state or basic chemistry class that diamond is tetrahedral. If you didn't know these, you could reason it out from other facts. Diamond is comprised solely from carbon which has four valence electrons out of 8 required for a full shell ($1s^2 2s^2 2p^2$), so carbon bonds with four other carbon atoms. This is likely to be a tetrahedral configuration in order to be stable.



Thus the answer is D.

44. Another random titbit of information, the attraction is due to phonons in the ionic lattice. Thus the answer is D.

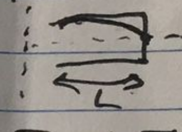
45. You should know the Doppler equation is $f=\left(\frac{v+v_r}{v-v_s}\right)f_0$, but also you must also know the meaning of each variable. In this case, v_r is the speed of the receiver <u>relative to the medium</u>, and v_s for the source, etc. As such, there is actually no Doppler effect in ~~the~~ this question. Thus the answer is C.

46. First of all, it is essential to identify the problem. With 1 speaker enclosure, this is an example of single slit diffraction. As such, the equation for the minima (~~for~~ i.e. destructive inference for the frequency to disappear) is $d\sin\theta = m\lambda$. The first minimum is at $m=1$

$\Rightarrow d\sin\theta = \frac{c}{f} \Rightarrow f = \frac{mc}{d\sin\theta} = \frac{350}{0.14\sin 45} = \frac{350}{0.14\cdot\left(\frac{1}{\sqrt{2}}\right)} \sim 3500\ Hz$

Thus the answer is D.

47. From the diagram, it is clear for a closed pipe that $\lambda = 4L$.



Thus $f_n = n\frac{v}{\lambda} = \frac{nv}{4L}$ where n is odd. Why is n odd?

$n=1 \Rightarrow 131\ Hz$, $n=3 \Rightarrow 393\ Hz$. Thus the answer is D.

48. First, look at inputs A and B. They are <u>inverted</u>, so we have $\bar{A}$ and $\bar{B}$. Then they enter a NOR gate, so we have $\overline{\bar{A}+\bar{B}}$. Looking at the options we are <u>done</u>, but as an exercise in notation, it is clear that we have $\overline{C\cdot D}$ from the NAND gate, and $\overline{\bar{A}+\bar{B}}\cdot\overline{C\cdot D}$ from the AND gate. Thus the answer is C.

49. For free atoms, the answer is a gas laser. Thus the answer is B.

50. $E_n = \frac{mZ^2(e^2)^2}{2\hbar^2(4\pi\varepsilon_0)^2 n^2} \propto \frac{mZ^2(e^2)^2}{n^2}$. Fear not, for dimensional analysis also solves this.

Thus the answer is C.

51. I is correct, and we see this thinking of the Bohr model, considering how emission lines of different elements look, etc. II is in contradiction to I, and so is incorrect. More formally, $E=\frac{-13.6Z^2}{n^2}$ indicates that energy transitions are fixed. III is also correct, as low temperature atoms are at the ground state. Thus the answer is D.

52. This is a case of Bragg scattering, where $d\sin\theta = \frac{n\lambda}{2}$. The clue for Bragg scattering is both the references to X-rays and crystals. Rearrange for d and plug in to get $d=\frac{m\lambda}{2\sin\theta} \approx 0.5\ nm$. Thus the answer is C.

53. Note that by $L_1=L_2 \Rightarrow m_1r_1v_1 = m_2r_2v_2$, where $v=\frac{2\pi r}{T}$

$\Rightarrow L = \frac{2\pi R^2 m_1}{T_1} = \frac{2\pi R^2 m_2}{T_2}$. Substitute $T_1 = 3T_2$ to find $\frac{m_1}{m_2}=3$

Thus the answer is D.

54. Note that the mass is unchanged, so there is no change in the force $F=\frac{GMm}{r^2}$ exerted on the planets. Furthermore, all planets are much greater than 3000m from the centre of the Sun, so they are not caught in the event horizon. Thus the answer is E.

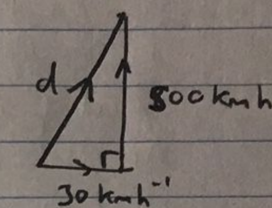
55. $\frac{\lambda}{\lambda_0} = \sqrt{\frac{1+\beta}{1-\beta}} \Rightarrow \frac{1+\beta}{1-\beta} = \left(\frac{580}{434}\right)^2 \approx \left(\frac{4}{3}\right)^2 \Rightarrow 9+9\beta = 16-16\beta$

$\beta = \frac{v}{c} = \frac{7}{25} \Rightarrow v = 0.28c$

Thus the answer is A.

56. A deceptively simple question that is about algebra rather than vectors.
By the diagram on the right, $d = \sqrt{800^2+30^2}$

$$t = \frac{v}{d} = \frac{500}{\sqrt{200^2 - 30^2}} = \frac{10 \cdot 50}{\sqrt{100}\sqrt{400-9}} = \frac{50}{\sqrt{391}}$$



Thus the answer is D.

57. The acceleration is $F = (2m+m)a \Rightarrow a = \frac{F}{3m}$

Thus $F_1 = ma = m\left(\frac{F}{3m}\right) = \frac{F}{3}$

and $F_2 = 2ma = 2m\left(\frac{F}{3m}\right) = \frac{2F}{3}$ Thus the answer is B.

58. This is simply

$F = m_B a = 10 \cdot 2 = 20N$ as $F_{NET} = F_{static}$

Thus the answer is A.

59. To derive the expression (if you know how) would be madness on the GRE. Let's use limiting cases to find the answer, noting that as $a \to 0$, $T \to 2\pi\sqrt{l/g}$ as expected.

(A) – wrong, we expect to see a difference as we're in a non-inertial frame!
(B) – as $a \to g$, $T \to \infty$ which is wrong (shouldn't blow up in free fall)
(C) – seems fine.
(D) –
(E) –

Alternatively, consider that the normal acceleration due to gravity is g, and so an acceleration upwards should yield g+a as the new acceleration, and thus $T = 2\pi\sqrt{\frac{l}{g+a}}$. Thus the answer is C.

61. Using the Lorentz Force law: $F = q\vec{E} + q\vec{v}\times\vec{B} \Rightarrow F = qvB$ as $\theta = 0$. Then, from centripetal force: $F = \frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB}$, where $r = d/2$. If q/m is doubled, then m/q is halved, so r is halved, and so d is halved. Thus the answer is E.

60. From $B = \frac{\mu_0 I}{2\pi x}$

$B_{45°} = \frac{\mu_0 I}{2\pi x \sin 45} = \frac{2\sqrt{2}\mu_0 I}{2\pi x}$

$B_{0°} = \frac{\mu_0 I}{2\pi x}$

$\Rightarrow B = \frac{\mu_0 I}{2\pi x}(1+2\sqrt{2})\hat{y}$

Thus the answer is C.

62. $\int E \cdot dA = \frac{Q}{\varepsilon_0} = \Phi_{TOT}$

$\Rightarrow \frac{1\times10^{-9}}{8.85\times10^{-12}} + 100 = 200\ Nm^2C^{-1}$

Thus the answer is E.

63. The reaction shown is ~~from~~ β^+ decay ($n \rightarrow p + e^+ + \nu_e$). This is an example of the weak interaction (as there is a change in quark flavour: the neutron udd becomes proton's uud). Either way you should realise that (A),(B),(C) make absolutely no sense. Thus the answer is D.

64. Recall that $L^2 Y_l^m(\theta,\phi) = l(l+1)\hbar^2 Y_l^m(\theta,\phi)$, so from $L=\sqrt{2}\hbar$ $\Rightarrow (\sqrt{2}\hbar)^2 = l(l+1)\hbar^2 \Rightarrow l=1$. Next recall that $L_z Y_l^m(\theta,\phi) = m\hbar Y_l^m(\theta,\phi)$. From $l=1 \Rightarrow m=-1,0,1$, so $L_z = -\hbar, 0, \hbar$. Even if you forget the last equation, $l=1$ should clue you in that there are 3 answers. Thus the answer is D.

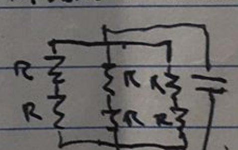
65. This is a good question to test deep understanding. I is correct, as $E_n = (n+\frac{1}{2})\hbar\omega$ (for 1D, or $E_n = (n+\frac{3}{2})\hbar\omega$ for 3D, etc), which is clearly evenly spaced. II is incorrect, as $U = \frac{1}{2}m\omega^2 x^2$. III is not correct as $\langle T\rangle = \langle U\rangle = \frac{E_n}{2} = \frac{(n+\frac{1}{2})\hbar\omega}{2} \neq 0$. IV is correct. Thus the answer is C.

66. $E_n \propto \frac{\mu}{n^2}$ where $\mu = \frac{M_\mu M_p}{M_\mu + M_p}$. Then, $E_n = \frac{-E_0}{n^2 \mu_H}\mu$
$\Rightarrow E_n = \frac{E_0}{n^2}\left(\frac{M_e+M_p}{M_e M_p}\right)\left(\frac{M_\mu M_p}{M_\mu + M_p}\right)$ Thus the answer is D.

67. Recall that $E = \frac{Q}{A\varepsilon_0}$. Then $\frac{dE}{dt} = \frac{1}{A\varepsilon_0}\frac{dQ}{dt}$ where $\frac{dQ}{dt} = I = 9A$. Then, the answer is $4\times10^{12}\ \mathrm{V\,m^{-1}\,s^{-1}}$

Thus the answer is D.

68. Note by symmetry that there is no current through the middle horizontal resistors. Thus $\frac{1}{R_{TOT}} = \frac{1}{R+R} + \frac{1}{R+R} + \frac{1}{R+R}$
$\Rightarrow R_{TOT} = \frac{2R}{3} \Rightarrow I = \frac{3V}{2R}$



Thus the answer is D.

69. $Z_{TOT} = Z_R + Z_C = R + \frac{1}{i\omega C}$. Then from ~~the~~ $V = IZ$, $I = \frac{V_{in}}{R + \frac{1}{i\omega C}}$
From this; $V_{out} = IZ_C = \frac{V_{in}}{R+\frac{1}{i\omega C}}\left(\frac{1}{i\omega C}\right) \Rightarrow G = \frac{V_{out}}{V_{in}} = \frac{1}{i\omega CR + 1}$
Thus the answer is D.

70. Recall that $\Delta\Phi_B = B\cdot\Delta A \Rightarrow \mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = \frac{-B\Delta A}{\Delta t}$. Also, $\mathcal{E} = IR$ $= \frac{\Delta q}{\Delta t}R \Rightarrow \Delta q = -\frac{B\Delta A}{R} = 10^{-4}\ \mathrm{C}$
Thus the answer is A.

71. $\frac{mv^2}{r} = qvB \Rightarrow v \propto R \Rightarrow \frac{v_1}{v_2} = \frac{R_1}{R_2} = \frac{1}{3}$
Thus the answer is B.

72. The answer is D. Here's how I remember it. I know fermions obey the Pauli exclusion principle, meaning two fermions cannot occupy the same state. This means $\psi_s(x_1, x_2) = -\psi(x_2, x_1)$ s.t. $\psi(x_1, x_2) - \psi(x_2, x_1) = 0$, and so they are antisymmetric. Technically this is derived the other way, but oh well. Bosons are symmetric, and so the Pauli exclusion principle doesn't apply. Thus the answer is D.

73. The J/ψ meson is $c\bar{c}$ (charm quark and antiquark). This is a good clue.
Thus the answer is D.

74. For a spherical mirror, $f = \frac{R}{2}$. Using $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$

$\Rightarrow \frac{1}{s'} = -\frac{2}{R} - \frac{1}{R} \Rightarrow s' = -\frac{R}{3} \Rightarrow d = \frac{R}{3}$ to the right

Thus the answer is E.

75.

Thus the answer is E.

76. Recall Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Looking at the interface from the air to the glass: $n_{air} \sin\theta = n \sin\theta_g$ where $n_{air} = 1$. There is also total internal reflection in this case

$\Rightarrow \theta = \arcsin\left(\sqrt{n^2 - 1}\right)$

Thus the answer is B.

77. $v_{rms} = \sqrt{\frac{3kT}{m}}$, and $t_{avg} = \frac{l}{v} \Rightarrow t_{avg} = l\sqrt{\frac{m}{3kT}}$

$\Rightarrow t_{avg} \propto \sqrt{m}$

Thus the answer is C.

78. $P(n) = \frac{e^{-\beta E_n}}{Z} = \frac{e^{-\beta E_n}}{\sum_i e^{-\beta E_i}}$ (for the canonical ensemble)

$p(n=2) = \frac{e^{-\beta E_2}}{e^{-\beta E_1} + e^{-\beta E_2}}$ where $\beta = \frac{1}{k_B T}$

Thus the answer is B.

79. $P = \frac{RT}{V-b} - \frac{a}{V^2} \Rightarrow W = \int_{V_1}^{V_2} \left(\frac{RT}{V-b} - \frac{a}{V^2}\right) dV \Rightarrow RT\int_{V_1}^{V_2} \frac{dV}{V-b} - a\int_{V_1}^{V_2} \frac{dV}{V^2}$

$= RT \ln\left(\frac{V_2 - b}{V_1 - b}\right) - a\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$

(D)

81. The initial $E_{TOT} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ where $I = \frac{1}{2}mR^2$.
~~and final E~~

Thus $\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 = mgh$. By the no-slip condition,

$\omega = \frac{v}{r} \Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = mgh$

$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = mgh$

$\Rightarrow h = \frac{3v^2}{4g}$

Thus the answer is B.

82. Remember $L = T - U$. Clearly from the answer choices $T = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2$, and this can be shown more formally by converting $\frac{1}{2}m(\dot{x}^2+\dot{y}^2)$ into polar coordinates as follows:

$x = r\cos\theta,\ y = r\sin\theta \Rightarrow \dot{x} = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt}$

$\Rightarrow \dot{x} = -r\sin\theta \cdot \dot{\theta}$ and likewise $\dot{y} = r\dot{\theta}\cos\theta$.

Then $\frac{1}{2}m(\dot{x}^2+\dot{y}^2) = \frac{1}{2}m\left(r^2\dot{\theta}^2\sin^2\theta + r^2\dot{\theta}^2\cos^2\theta\right)$

$= \frac{1}{2}mr^2\dot{\theta}^2(\sin^2\theta + \cos^2\theta) = \frac{1}{2}mr^2\dot{\theta}^2 =$ rotational

U is just the elastic potential, typically $\frac{1}{2}kx^2$. Here, $x = r - s$.

Thus the answer is D.

83. The Hamiltonian is a function of θ, p_θ, and p_ϕ. Noting that $\dot{p}_\phi = -\frac{\partial H}{\partial \phi} = 0$, p_ϕ is constant.

Thus the answer is E.

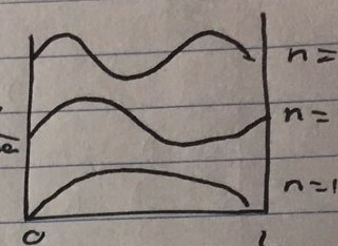
84. No need to calculate here! If the density is ~~decreasing~~ increasing with length ($\lambda \propto x$), then the majority of the mass is closer to L than $x=0$. Thus $m > \frac{1}{2}L$, giving $\frac{2}{3}L$ as the only option.
For formality, though:

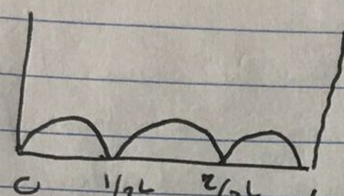
$x_{cm} = \frac{1}{M}\int_0^L \rho(x)\,dx = \frac{1}{M}\int_0^L (x\lambda)\,dx = \frac{1}{M}\int_0^L \frac{2M}{L^2}x^2\,dx$

$= \frac{2L}{3}$ $\square$

Thus the answer is E.

85. The general equation for the wave function of a particle in an infinite square well is $\psi(x) = A \sin\left(\frac{n\pi x}{L}\right)$; here $n=3$. You should know that this corresponds to the number of antinodes n (and thus $n-1$ nodes). Now, $P(x) = |\psi(x)|^2$ so the probability distribution for $n=3$ is:





The antinodes are equally spread, so $1/3$ of the area is between $x = 1/3 L$ and $x = 2/3 L$.

Thus the answer is B.

86. First of all, the eigenvalues of a Hermitian matrix are real, so (D) and (E) are wrong. Set up the characteristic equation

$$\det\begin{pmatrix} 2-\lambda & i \\ -i & 2-\lambda \end{pmatrix} = 0 \Rightarrow (2-\lambda)(2-\lambda) - i(-i) = 0$$

$$\Rightarrow \cancel{4-4\lambda-1=0} \Rightarrow 4 - 4\lambda + \lambda^2 - 1 = 0$$

$$\Rightarrow \lambda^2 - 4\lambda + 3 = 0 \Rightarrow (\lambda-3)(\lambda-1) = 0$$

$\lambda = 1, 3$ Thus the answer is B.

87. $$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

~~as expected, th~~

$$= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix}$$

$$= 2i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = 2i\sigma_z.$$ Thus the answer is D.

88. First, normalise: $|A|^2 \begin{pmatrix} i-1 & 2 \end{pmatrix}\begin{pmatrix} i+1 \\ 2 \end{pmatrix} = ((i-1)(i+1) + 4]|A|^2 = 1$

$\Rightarrow |A| = \frac{1}{\sqrt{6}}$. Now $\chi = \frac{i+1}{\sqrt{6}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{2}{\sqrt{6}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\left(= \frac{1}{\sqrt{6}}\begin{pmatrix} i+1 \\ 2 \end{pmatrix}\right)$.

Then $P\left(S_z = -\frac{\hbar}{2}\right) = \left(\frac{2}{\sqrt{6}}\right)^2 = \frac{2}{3}$

Thus the answer is D.

89. Such questions are common, and are always solved by inspection of the answer choices. We are looking for an answer where if $k_1 = k_2$, there is no step potential, and no reflection. i.e.: $k_1 = k_2 \Rightarrow |R| = 0$. This is $R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$

Thus the answer is D.

90. Note this is asking about V and not E, a common mistake. Remember that potentials are additive, so region II is $\cancel{\frac{1}{4\pi\varepsilon_0}\frac{Q}{b}} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}$

Thus the answer is D

91. A big clue here is that this is the electrostatic potential, i.e. for $\vec{B}=0$. Then $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ reduces to $\nabla\times\vec{E}=\vec{0}$. Thus the answer is C.

92. Let us cancel down options. ~~From~~ There is no ~~charge~~ current at $0<r<R$, and thus by ~~Gauss's~~ Ampere's Law, $B=0$, so B and C are out. Now, magnetic fields are continuous across a surface

Thus the answer is E.

93. Remember that for a dielectric, all equations hold with ε in place of ε_0, where $\varepsilon = k\varepsilon_0$. Thus the new equations are $U=\frac{1}{2}CV^2$ where ~~$C=\frac{V}{q}$ (with no use of $\varepsilon/\varepsilon_0$), so~~ $\Rightarrow U=U_0$, and $C=\frac{\varepsilon A}{d}$

$\Rightarrow C = \frac{k\varepsilon_0 A}{d} = kC_0$, and so $U=\frac{1}{2}kC_0V$

$\Rightarrow U = kU_0$, and $E=\frac{\sigma}{\varepsilon} = \frac{\sigma}{k\varepsilon_0} \Rightarrow E = \frac{E_0}{k} = \frac{V}{kd}$

$\Rightarrow E = \frac{V}{kd}$ Thus the answer is E.

94. Using the Lorentz transformations, $\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$ and

$\Delta x' = \gamma(\Delta x - v\Delta t) \Rightarrow \Delta x = - \frac{c^2\Delta t'}{\gamma v}$

$|v| = c^2\left|\frac{\Delta t'}{\Delta x'}\right| = 0.36c$

Thus the answer is C

95. Note the relation $[AB,C]=A[B,C]+[A,C]B$.

Then $[J_xJ_y,J_x] = J_x[J_y,J_x]+[J_x,J_x]J_y$.

Now $[J_y,J_x] = -[J_x,J_y] = -i\hbar J_z$, and $[J_x,J_x]=0$,

so $[J_xJ_y,J_x] = i\hbar J_z$.

Thus the answer is B.

96. n-type = negative: there exists an excess of electrons from replacing germanium (with 4 valence electrons) with a dopant with more valence electrons. Recalling your periodic table knowledge, Boron (B) has only 3 valence electrons, and so would be used for a p-type semiconductor, not an n-type one

Thus the answer is D.

97. In Compton scattering $\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$. At $\theta=90^\circ$,

$\Delta\lambda = \frac{h}{mc} \Rightarrow \lambda-\lambda_0 = \frac{hc}{E} - \frac{hc}{E_0} = \frac{h}{mc}$

$\Rightarrow E = \frac{E_0 mc^2}{E_0+mc^2}$

Thus the answer is E

98. ~~Going~~ Through conservation rules: (A) - breaks lepton number, (B) - breaks baryon number, (C) - doesn't conserve muon lepton number, (E) doesn't conserve electron lepton number.

breaks number

Thus the answer is D.

(D)	μ^+	$\rightarrow$	e^+	$+\, \nu_e$	$+\, \bar{\nu}_\mu$
charge	+1	$\rightarrow$	+1	0	0
baryon	0	$\rightarrow$	0	0	0
lepton:	1	$\rightarrow$	1	0	0
muon:	$\cancel{1}$	$\rightarrow$	0	+1	0
electron:	0	$\rightarrow$	-1	0	+1

99. $a_{rad} = \frac{v^2}{r} = \omega^2 r$, and $a_{tan} = \frac{dv}{dt} = \frac{d(\omega r)}{dt} = r\frac{d\omega}{dt} = r\alpha$

then, $\tan(\theta) = \frac{a_{tan}}{a_{rad}} \Rightarrow \theta = \arctan\left(\frac{\alpha}{\omega^2}\right)$

Thus the answer is E.

100. $Z = \sum_n' e^{-\beta E_n} = e^{-\frac{\beta \hbar \omega}{2}} \sum_n e^{-\beta n \hbar \omega} = 1 + e^{-\beta \hbar \omega} + e^{-2\beta \hbar \omega} + \dots$

By knowledge of converging infinite geometric series:

$S_\infty = \frac{a}{1-r} \Rightarrow Z = e^{\frac{-\hbar \omega}{2kT}} \left(\frac{1}{1 - e^{-\hbar\omega/kT}}\right) \quad \left[\beta = \frac{1}{kT}\right]$