Draft version January 7, 2020
Typeset using LaTeX **modern** style in AASTeX61

MODELLING THE SURFACE TEMPERATURES OF EXOPLANETS

Samuel Moore[1]

[1]*Department of Astrophysical Sciences, Princeton University*

Abstract

When designing a General Circulation Model (GCM) for an exoplanet, there are a multitude of different factors that must be considered to create an accurate climatic model, with primary factors being stellar irradiance, atmospheric composition, and atmospheric thickness. Other factors include rotational frequency and tidal locking in combination with climatic feedback mechanisms. This paper seeks to investigate one aspect of GCMs: the modelling of surface temperatures. The paper begins by building a simple model for calculating the surface temperature of a planet, and then improves upon the model by considering the effect of atmospheric composition upon the optical depth of various hypothetical atmospheres, including 100% H_2, N_2, O_2, O_3, CO_2, H_2O, and CH_4 atmospheres, as well as various mixtures of these compounds. In addition the Earth's atmosphere is modelled to a reasonable degree of accuracy, and further suggestions are made as to how the model could be improved upon. This temperature model is then used to give an approximation to the boundaries of the habitable zone given various properties of the planet, including mass and atmospheric composition.

Keywords: GCM, exoplanet, atmosphere, habitable zone

1. INTRODUCTION

The term exoplanet simply refers to any planet (as defined by the IAU in 2006) that is beyond our solar system. As a result, there are many different ways to classify such planets. One classification system proposed by the Planetary Habitability Laboratory (PHL) for the University of Puerto Rico involves comparing the exoplanets to well-known planets in the Earth's solar system based on the mass of the exoplanet. For example, gas-giant exoplanets with short orbital periods can be classified as Hot Neptunes for 10-50 Earth masses (M_E), such as Gliese 436b (Butler et al, 2004), or Hot Jupiters in the range of 50-5000 M_E, such as Kepler 7-b (Demory et al., 2011), while other classifications include Earths (0.5-2 M_E) and super-Earths (2-10 M_E). A more general but nevertheless useful classification is a division between terrestrial planets and gas-giant planets. Due to the fundamental differences between these types of exoplanets, such as atmospheric composition, the distinction is important in the development of General Circulation Models (GCMs). A GCM refers to a computational model that seeks to numerically model the climate of a planet. Various GCMs consider a range of factors; the most important factors include the atmosphere (temperature, water vapour distribution, circulation), oceans (temperature, salinity levels and circulation patterns), and terrestrial processes (including greenhouse gas absorption). Because exoplanets vary widely in these factors, GCMs may be tailored specifically to the planets they are studying. Building a comprehensive GCM would be beyond the scope of the paper. This paper seeks to develop a simple model for investigating the different factors that affect the surface temperature of a planet, and then compares the results with the scientific literature that uses more advanced models. It is important to note that the idea of a surface is only applicable to terrestrial planets, as gas giants do not have solid surfaces. As a result, this paper will investigate surface temperature for planets at a maximum of 15 M_E to include the analysis of super-Earths.

1.1. *Equilibrium Temperature*

To begin with the analysis of GCMs and their design, it is a useful exercise to take a theoretical, simplified model, apply it to well-studied planets such as the Earth, and then increase the complexity of the model until it can deal with a range of factors. By considering energy balance, it is possible to derive an equilibrium temperature for a blackbody planet, as various authors have already done (e.g: Kaltenegger, 2017). Consider that for a planet at equilibrium temperature, there is a balance of energy: $E_{absorbed} = E_{radiated}$. Given that there is sufficient atmospheric circulation such that latitudinal temperatures are approximately constant, then energy is radiated from the whole planet equally over 4π steradians. From the Stefan-Boltzmann Law, the energy emitted per unit time is given by: $P_{radiated} = A\epsilon\sigma T^4$. Assume that the planet is a blackbody so that $\epsilon = 1$. Meanwhile, considering a unitary star system with radiation being absorbed on only half of a planet at a time $P_{absorbed} = S(1-A)\pi R^2$,

Planetary Properties					
Planet/Body	Stellar Flux (Wm^{-2})	Bond Albedo	Calculated $T_{eq}(K)$	Actual $T_{surf}(K)$	Difference (K)
Mercury	9082.7	0.068	439.5	440	0.5
Venus	2601.3	0.770	226.6	733	503.4
Earth	1361.0	0.306	254.3	288	33.7
Mars	586.2	0.250	209.8	218	9.2

Table 1. By applying Equation 1, a theoretical equilibrium temperature can be found for a range of planets. By comparison with the actual average temperature of the planet sourced from NASA fact sheets, the temperature difference can be calculated. The large differences show that the initial model is inadequate to predict temperatures of bodies, primarily because the greenhouse effect has been ignored. These values are also confirmed in other scientific papers, such as Kaltenegger, 2017.

where A is the Bond albedo of the planet, and S is the stellar flux. By setting these equations equal to each other and rearranging:

$$T_{equilibrium} = \sqrt[4]{\frac{S(1-A)}{4\sigma}} \tag{1}$$

The stellar flux, S, can be calculated by modelling the star in question as a blackbody, calculating the total radiation emitted by the star, and then using this to calculate the stellar flux.

$$S = \frac{P}{4\pi d^2} = \frac{R^2 \sigma T^4}{d^2} \tag{2}$$

Thus, there are two variables in this simplified model for finding the equilibrium temperature of a body: Bond albedo, and stellar flux. The Bond albedo is defined as the fraction of solar radiation that is reflected back to space through the top of the atmosphere (Palle et al., 2016). For example, according to NASA, the Earth has an overall bond albedo of 0.303 (Williams, 2004), and the Sun-Earth value for stellar flux is 1361 Wm^{-2}, which is verified by several scientific sources, (e.g: Coddington, 2016).

1.2. *Greenhouse Effect*

The primary reason for the difference in the expected temperature and the actual temperature is due to the effect of greenhouse gases (GHGs) in the atmosphere. A GHG is a gas that absorbs and emits IR radiation, resulting in the greenhouse effect. Key examples of GHGs for the Earth include $H_2O, CO_2, O_3, CH_4, N_2O$, and various chlorofluorocarbons (CFCs). The greenhouse effect occurs because short-wave radiation from a star can pass through the atmosphere with minimal absorption, and then is absorbed by the Earth. The electromagnetic (EM) wave is then re-emitted with slightly less energy, becoming long-wave IR radiation which is absorbed by the atmosphere. The absorption spectrum of GHGs varies depending on the gas, and so

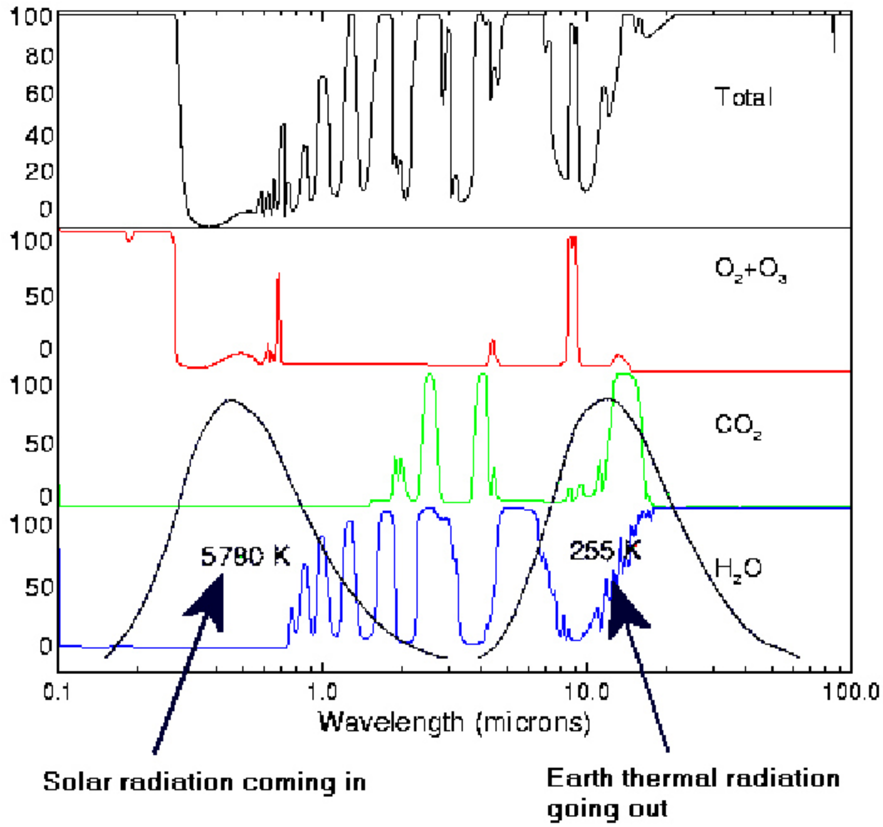

Figure 1. A figure showing the absorption spectra of several greenhouse atmospheric gases that are significant in the Earth's atmosphere. Superimposed onto this are the black body EM emission curves for the Sun and the Earth. There is little overlap between the absorption lines and the Sun's emission curve in comparison to the Earth's emission curve, showing that there is far more absorption when EM waves are emitted from the Earth, compared to EM waves emitted from the Sun. (Source: https://www.cabrillo.edu)

different GHGs will lead to different degrees of warming. An illustration of how the greenhouse effect works is shown in Figure 1.

The greenhouse effect can be observed with planets in the solar system. For example, Venus has a atmosphere with a very high concentration of GHGs relative to the Earth, at 96.5% CO_2 for Venus (Basilevsky and Head, 2003), resulting in a higher surface temperature difference, while Mercury has a minimal atmosphere (Domingue et al, 2007) resulting in a lower difference, as shown in Table 1. This explains why atmospheric composition is integral to the climate modeling of certain exoplanets, as well as factors that may affect this composition over time, such as geological activity, as well as the presence of an ocean which could increase water vapour and influence other climate feedback mechanisms.

1.3. *Optical Depth*

In order to consider the greenhouse effect, it is necessary to account for optical depth, τ. Optical depth is a function of the opacity of a material with respect to distance through the medium (in the case of an atmosphere, downwards from the top). It is important to note that there are differences between the nature of optical depth for a star compared to optical depth for an atmosphere. When analysing a stellar envelope, there are 4 primary causes of optical depth:

1. Free-free - caused when a free electron is absorbed by a nucleus. This is most important at high temperatures when there are lots of free electrons.

2. Bound-free - where a photon ionizes an atom, causing the electron and the ion to repel each other.

3. Bound-bound - caused by atoms absorbing photons and becoming excited.

4. Electron Scattering = specifically Thomson scattering. It should be noted that this component of optical depth is insignificant unless at high temperatures.

However, when studying an atmosphere, there is no ionisation of the gas molecules because the temperature is much lower. As a result, only bound-bound scattering is important for atmospheric optical depth.

Using the Eddington approximation to the Milne problem, it is possible to calculate the surface temperature as a function of both the effective temperature, and the optical depth, τ, of the atmosphere, where the optical depth will vary based on the composition of the atmosphere:

$$T_{surface} = \sqrt[4]{\frac{3}{4}T_{eff}^4(\frac{4}{3}+\tau)} \tag{3}$$

Where $T_{eff} = T_{eq}$. The equation is adapted so that $T_{surface} = T_{eff}$ when $\tau = 0$. This equation holds under the assumption of a plane-parallel grey atmosphere. The optical depth will then vary depending on the specific composition and density of the atmosphere. The next step is to calculate the optical depth for different atmospheres. in order to then model a hypothetical atmosphere that consists of just one gas, such as N_2, or CO_2.

The optical depth of the atmosphere, defined as $\tau = \int_0^z \kappa_\lambda \rho(z)dz$. The density ρ of a gas is a function of both temperature and pressure, and so will vary with altitude. From the hydrostatic equation $\frac{dP}{dz} = -g\rho$, one can derive $P = P_o e^{-\frac{z}{H}}$ and $\rho = \rho_o e^{-\frac{z}{H}}$ where H is the scale height $H = \frac{RT}{mg}$, with m being the mean molecular mass of one atmospheric particle in $kg\ mol^{-1}$. From this,

$$\tau = \kappa_\lambda H \rho_o = \frac{\kappa_\lambda \rho_o RT}{mg} \tag{4}$$

Given that g is a function of M_p and R_p (the mass and radius of the planet respectively), it is necessary to introduce the equation $g = \frac{GM_p}{R_p^2}$. Then, substituting the optical depth into equation 1, we find the following equation for $T_{surface}$.

$$T_{surface} = \sqrt[4]{\frac{3}{4}T_{eff}^4(\frac{4}{3} + \frac{\kappa_\lambda \rho_o RTR_p^2}{GM_p m})} \tag{5}$$

It should be noted that because constant density cannot be assumed for planets of varying mass (especially super-Earths), it is necessary to derive a power relationship between the mass of a planet and its radius. Using the values for the Earth, $M_{earth} = 6.02 * 10^{24} kg$ and $R_{earth} = 6.37 * 10^6 m$, it can be approximated that for Earth-like planets, $R_p \approx M_p^{0.27.46}$. From this:

$$T_{surface} = \sqrt[4]{\frac{3}{4}T_{eff}^4(\frac{4}{3} + \frac{\kappa_\lambda \rho_o RT}{GmM^{0.451}})} \tag{6}$$

An approximate method to calculate τ is to use $\kappa_\lambda = \kappa_R$, the Rosseland mean opacity value, which is a function of the surface pressure and equilibrium temperature of the gas. Using the Rosseland mean opacity values given by the HITRAN database (given in part in Badescu, 2009), it is now possible to gauge initial estimates for the actual surface temperature of a planet, considering its atmospheric composition.

The Rosseland mean opacity is given by

$$\frac{1}{\kappa_R} = \frac{\int_0^\infty \frac{1}{\kappa_v} \frac{dB_v}{dT} dv}{\int_0^\infty \frac{dB_v}{dT} dv} \tag{7}$$

where

$$B_v = \frac{2hc^2/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1} \tag{8}$$

Alternatively, by considering the four primary causes of opacity given above, the opacity may be given as:

$$\kappa = \kappa_{\lambda,ff} + \kappa_{\lambda,bf} + \kappa_{\lambda,bb} + \kappa_{\lambda,es} \tag{9}$$

This can be averaged over the relevant range of wavelengths to give the Rosseland mean opacity value, with the values for κ_{ff}, κ_{bf}, and κ_{es} set at zero because of the low atmospheric temperature:

$$\kappa_R = \kappa_{bb} \tag{10}$$

It is possible to find the κ_R values for a variety of molecules at low temperatures using the HITRAN database. First of all, the opacity values can be outputted for specific isotopologues for a range of wavelengths (as shown in Figure 2), and then Equation 7 can be used to generate values for the Rosseland mean opacity. More details on the methods to calculate the Rosseland mean opacity can be found in Badescu, 2009. K_R values are given in Table 2 for different molecules. These values have been calculated using HAPI (HITRAN Application Programming Interface) for all molecules, using all listed isotopologues in the database for each molecule. These values are then checked against certain known κ_R values given by Badescu, 2009.

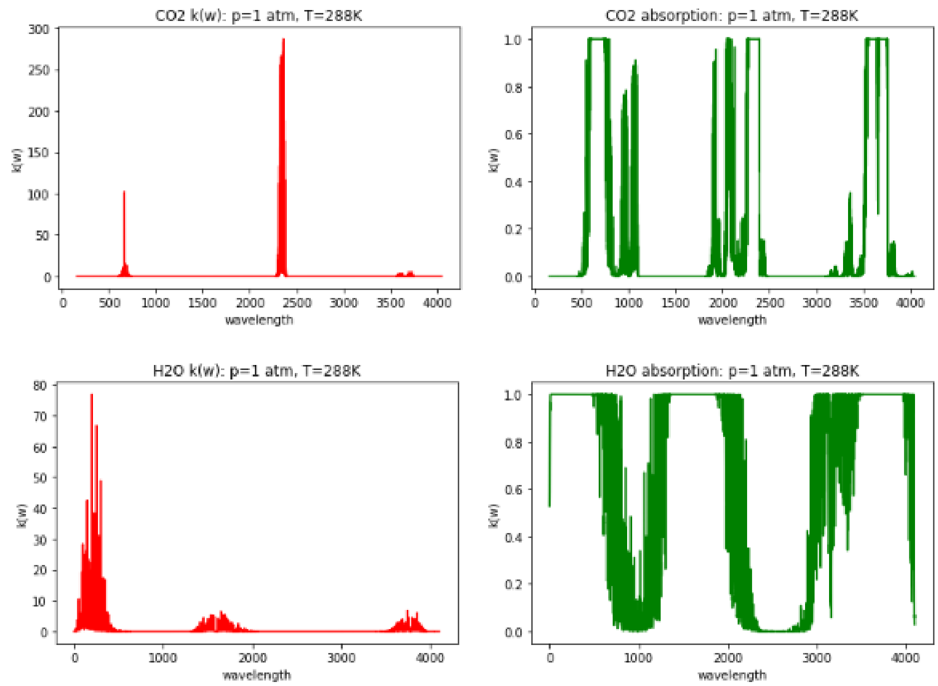

Figure 2. The above figures show the absorption spectra of various molecules at 288K, 1 atm, generated using a Voigt profile. All graphs were generated using HPAI (HITRAN Programming Application Interface). As shown, all molecules have a different absorption spectra. Using this data from HITRAN. it is then possible to find the Rosseland mean opacity values for specific temperatures and pressures, as given in Table 2.

As Badescu does not give exact κ_R values for $T = 288K$ and $P = 101kPa$, it was necessary to use linear interpolation to find K_R values for these parameters. This is unlikely to significantly affect calculated surface temperature values because linear interpolation did not change the order of magnitude of the κ_R value for any molecule. Upon comparing the HAPI values for N_2, CO_2, and CH_4 with Badescu, the values closely correspond.

It should be noted that κ_R is a function of both temperature and pressure, which appears problematic because both the temperature and the pressure vary with altitude, and the surface temperature is what is being calculated using κ_R. To begin with, set $T = T_{eq}$ so that a value for κ_R can be selected using the atmospheric surface pressure. Upon calculating a value for $T_{surface}$ from equation 4, this new value can be used in the equation as T, and also can be used to find the Rosseland mean opacity value. This will give a closer value for $T_{surface}$. After repeating this process several times, a more accurate value of $T_{surface}$ will be produced.

2. APPLICATION OF EQUATIONS

Using Equation 3 for planets with a known surface temperature allows for their expected optical depth to be calculated. For example, Earth has $\tau = 0.836$, whilst Mars has $\tau = 0.221$. By then using Equation 4, it is possible to calculate the Rosseland mean opacity value for the Earth's atmosphere as a whole, giving $\kappa_{earth} = 0.828 *$

Rosseland Mean Opacity Values					
Gas	Temperature (K)	Pressure (kPa)	κ_R ($m^2\ kg^{-1}$)	m ($kg\ mol^{-1}$)	Source
H_2	288	101	$0.151 * 10^{-3}$	0.002	HPAI
N_2	288	101	$0.136 * 10^{-11}$	0.028	HPAI, Badescu (2009)
O_2	288	101	$0.421 * 10^{-11}$	0.032	HPAI
O_3	288	101	$0.541 * 10^{-3}$	0.048	HPAI
CO_2	288	101	$0.136 * 10^{-1}$	0.044	HPAI, Badescu (2009)
H_2O	288	101	$0.796 * 10^{-3}$	0.018	HPAI
CH_4	288	101	$0.280 * 10^{-2}$	0.016	HPAI, Badescu (2009)
$Earth$	288	101	$0.828 * 10^{-4}$	0.029	Equation 4

Table 2. A table showing a selection of the calculated Rosseland Mean Opacity Values for temperature and pressure values used to approximate Earth-like planets. The values have been calculated using HITRAN Application Programming Interface (HPAI), and checked against κ_R values given by another scientific source (Badescu, 2009). The mass of each molecule, m is also given in $kg\ mol^{-1}$, as this is another variable specific to each molecule that affects the optical depth value.

$10^{-4} m^2 kg^{-1}$. This will be useful in investigating the scaling of Earth-like planets when factors such as the mass of the planet are changed.

Equation 4 can be adapted to calculate an average optical depth for atmospheres with more than one compound.

$$\tau = (f_1 \kappa_{R1} + f_2 \kappa_{R2} + ... + f_n \kappa_{Rn}) H \rho_o \tag{11}$$

where $f_1, f_2..., f_n$ is the mass fraction of the respective compound in the atmosphere. Note that κ_R is in $m^2 kg^{-1}$.

By use of Equation 6, the various factors that affect the surface temperature of a planet can be investigated. In this paper, different relationships are investigated graphically. First of all, the composition of the atmosphere is changed, while keeping the mass of the planet, M_p, the atmospheric surface density, ρ_o, and the effective temperature T_{eff} remain constant. Secondly, M_p is changed, while the atmospheric composition, ρ_o, and T_{eff} are kept constant. Thirdly, ρ_o is changed, with the atmospheric composition, M_p, and T_{eff} remaining constant. Finally, T_{eff} is indirectly varied by changing the albedo, the Star-planet distance, and the temperature of the star, while atmospheric composition, M_p, and ρ_o are kept constant. Note that κ_R and m are both determined by atmospheric composition.

To begin with, several hypothetical planets are considered with atmospheres that consist of only one compound: H_2, N_2, O_2, O_3, H_2O, CO_2, and CH_4. Basic analysis of Equation 6 demonstrates that increasing the mass of the planet decreases the optical depth (and the surface temperature) in a non-linear manner, when κ_R, m, T_{eff} and ρ_o are kept constant. Figure 3 shows the effect of varying the mass of a planet for different atmospheric compositions. In addition, if the atmospheric surface density ρ_o is increased, then the optical depth (and surface temperature) will increase in a non-linear manner. Figure 4 shows the effect of varying the atmospheric surface density for different atmospheric compositions.

2.1. *Hydrogen Exoplanet*

Consider a hypothetical planet that has a 100% hydrogen atmosphere, with all other conditions identical to that of the Earth. From this, it can be calculated that $\tau = 1.96$, and so $T_{surface} = 302K$. Figure 3 shows the effect of varying the mass of a 100% H_2 atmosphere planet, whilst Figure 4 shows the effect of varying the atmospheric surface density.

According to one paper, a pure H_2 atmosphere on a planet of $3M_E$ with a surface pressure of 40 bars can maintain a surface temperature of 280K when 1.5AU away from an early type M dwarf star, and 10AU away from a type G star. (Pierrehumbert and Gaidos, 2011). The model used by Pierrehumbert and Gaidos gives early type M dwarf stars a temperature of 3000K, and type G stars a temperature of around 6000K. From this, it is possible to calculate the surface temperature using Equation 6, and see how the surface temperature values compare.

Using Equation 6, an early type M dwarf star with a distance of 1.5 AU predicts $T_{surface} = 294K$, while a type G star at 10 AU predicts $T_{surface} = 292K$. These values are reasonably accurate when compared to the values given by Pierrehumbert and Gaidos.

In addition, a $N_2 - H_2$ atmosphere has been considered, with Figure 5 showing how varying the percentage of H_2 changes the surface temperature. As the composition changes, the Rosseland mean opacity and average molecular mass changes, resulting in the characteristic $N_2 - H_2$ curve.

2.2. *Nitrogen Exoplanet*

Consider a hypothetical planet that has a 100% N_2 atmosphere, with all other conditions identical to that of the Earth. It can be calculated that $\tau = 1.37 * 10^{-8}$, and so $T_{surface} = 255.0K$, when $T_{eff} = 255K$. As expected, this is lower than the surface temperature of the Earth, as GHGs have been replaced by a non-GHG. Figure 3 shows the effect of varying the mass of a 100% N_2 atmosphere planet, whilst Figure 4 shows the effect of varying the atmospheric surface density. The effect of a pure N_2 atmosphere on the surface temperature of exoplanets is clearly negligible.

2.3. *Oxygen Exoplanet*

For a hypothetical planet that has a 100% O_2 atmosphere, with all other conditions identical to that of the Earth, $\tau = 0.003$, and so $T_{surface} = 255.16K$. As expected, this is lower than the surface temperature of the Earth, as GHGs have been replaced by a non-GHG. Figure 3 shows the effect of varying the mass of a 100% O_2 atmosphere planet, whilst Figure 4 shows the effect of varying the atmospheric surface density.

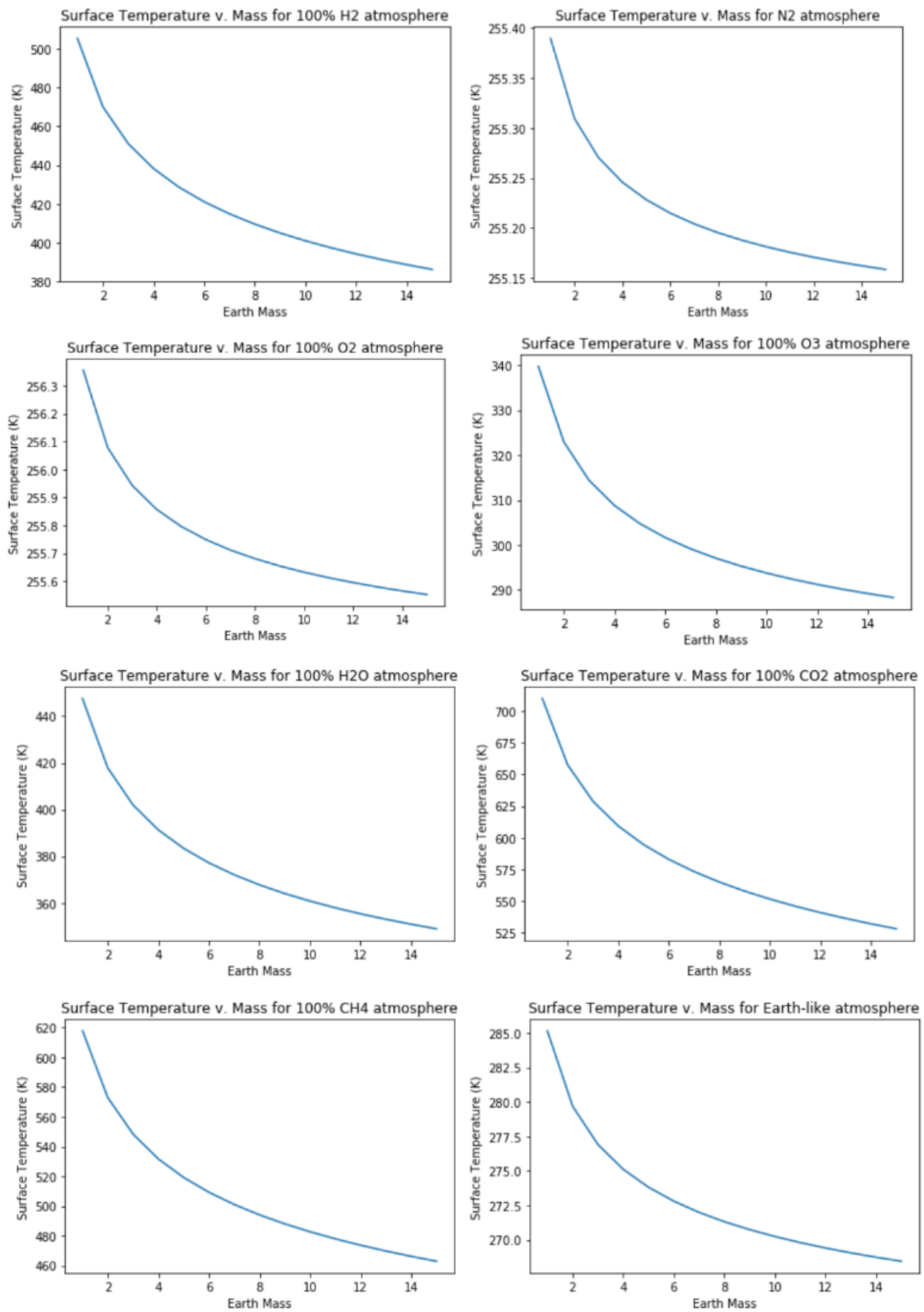

Figure 3. Plots of surface temperature vs. the mass of the planet using the Rosseland mean opacity value for various pure atmospheres, as well as the Earth's atmosphere. All other variables (T_{eff} and ρ_o) are kept constant, and are set at Earth values.

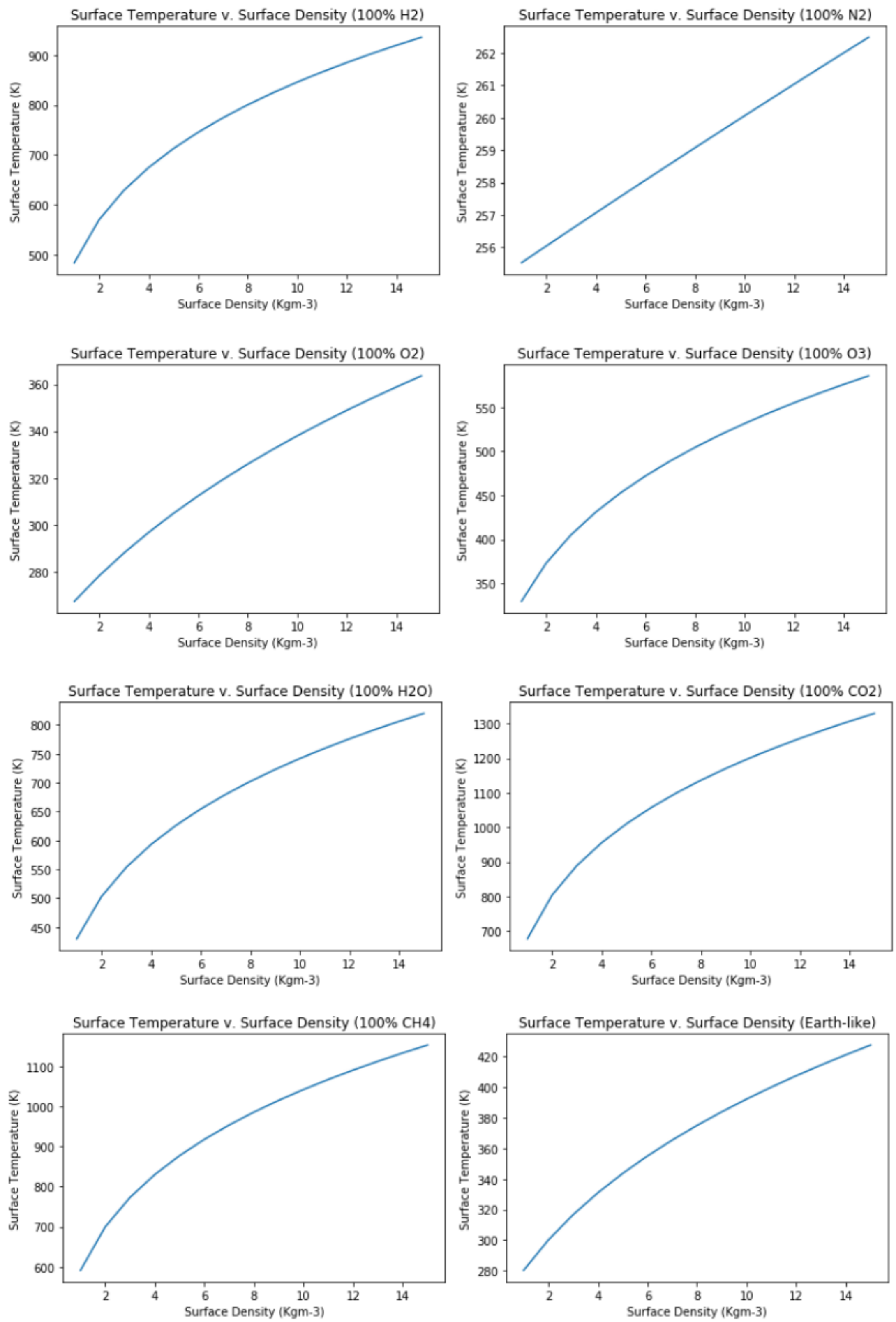

Figure 4. Plots of surface temperature vs. the atmospheric surface density of the planet using the Rosseland mean opacity value for various pure atmospheres, as well as the Earth's atmosphere. All other variables (T_{eff} and M_E) are kept constant, and are set at Earth values.

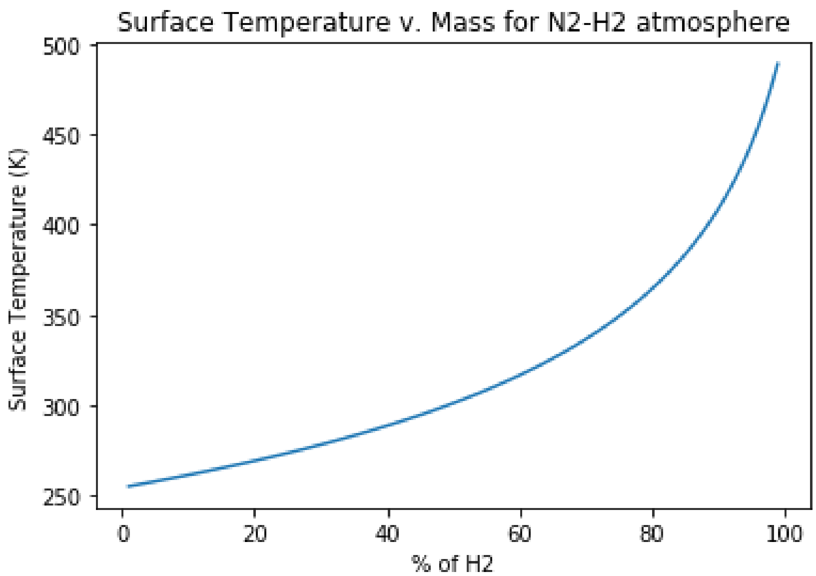

Figure 5. A graph showing the variation in surface temperature for a $N_2 - H_2$ atmosphere, with the percentage of H_2 varying.

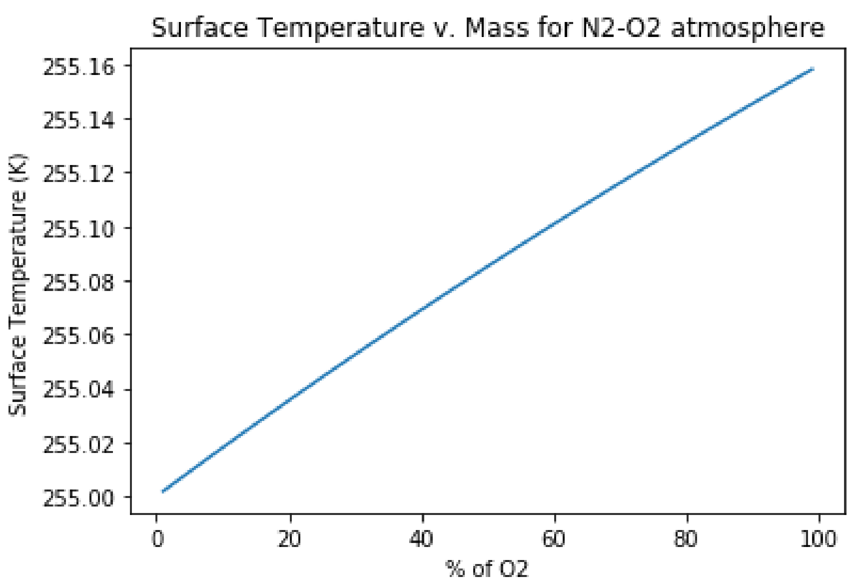

Figure 6. A graph showing the variation in surface temperature for a $N_2 - O_2$ atmosphere, with the percentage of O_2 varying.

In addition, a $N_2 - O_2$ atmosphere has been considered, with figure 6 showing how varying the percentage of O_2 changes the surface temperature. As the composition changes, the Rosseland mean opacity and average molecular mass changes, resulting in the $N_2 - O_2$ curve shown. As both N_2 and O_2 individually appear to have a minor effect on surface temperatures, varying the percentage of O_2 has a very small effect on changing the surface temperature.

2.4. *Ozone Exoplanet*

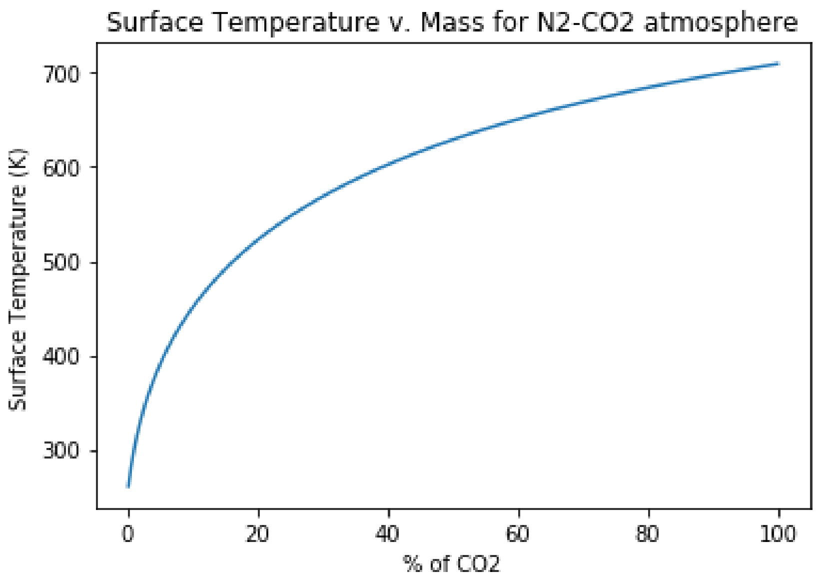

Figure 7. A graph showing the variation in surface temperature for a $N_2 - CO_2$ atmosphere, with the percentage of CO_2 varying.

For a hypothetical planet that has a 100% O_3 atmosphere, with all other conditions identical to that of the Earth, $\tau = 5.46$, and so $T_{surface} = 383.1K$. As expected, this is higher than the surface temperature of the Earth, as non-GHGs have been replaced by a GHG. Figure 3 shows the effect of varying the mass of a 100% O_3 atmosphere planet, whilst Figure 4 shows the effect of varying the atmospheric surface density.

2.5. *Carbon Dioxide Exoplanet*

Considering a hypothetical planet that has a 100% CO_2 atmosphere, with all other conditions identical to that of the Earth. It can be calculated that $\tau = 137.28$, and so $T_{surface} = 814.2K$. As expected, this value is much higher than the temperature of the Earth, as all non-GHGs have been replaced by a GHG. Figure 3 shows the effect of varying the mass of a 100% CO_2 atmosphere planet, whilst Figure 4 shows the effect of varying the atmospheric surface density.

In addition, a $N_2 - CO_2$ atmosphere has been considered, with Figure 7 showing how varying the percentage of CO_2 changes the surface temperature. As the composition changes, the Rosseland mean opacity and average molecular mass changes, resulting in the $N_2 - CO_2$ curve shown. It is clear that CO_2 can significantly raise the surface temperature of an $N_2 - CO_2$ exoplanet, with a greater rate of change in surface temperature seen at lower percentages of CO_2.

2.6. *Water Vapour Exoplanet*

Considering a hypothetical planet that has a 100% H_2O atmosphere, with all other conditions identical to that of the Earth, $\tau = 8.04$, and so $T_{surface} = 415.2K$. As expected, this value is much higher than the temperature of the Earth, as all non-GHGs have been replaced by a GHG. Figure 3 shows the effect of varying the mass

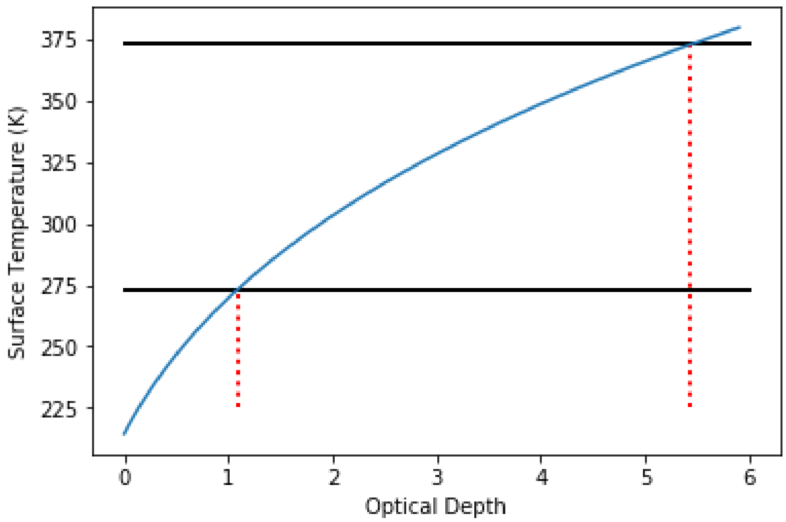

Figure 8. A plot of Surface Temperature vs. Optical Depth for $T_{eq} = 255K$. The black line shows the bounds for the habitable zone, $T_{lower} = 273K$, $T_{upper} = 373K$, giving bounds on the optical depth, $\tau_{lower} = 1.0849$, $\tau_{upper} = 5.4373$

of a 100% H_2O atmosphere planet, whilst Figure 4 shows the effect of varying the atmospheric surface density.

2.7. *Methane Exoplanet*

For a hypothetical planet that has a 100% CH_4 atmosphere, with all other conditions identical to that of the Earth, $\tau = 28.26$, and so $T_{surface} = 553.5K$. As expected, this value is much higher than the temperature of the Earth, as all non-GHGs have been replaced by a GHG. Figure 3 shows the effect of varying the mass of a 100% CH_4 atmosphere planet, whilst Figure 4 shows the effect of varying the atmospheric surface density.

2.8. *Earth-like Exoplanet*

Figure 3 also shows the effect of varying the mass of a planet with an Earth-like atmosphere and Figure 4 shows the effect of varying the atmospheric surface density. It is also of interest to consider how other factors may affect the surface temperature for a planet with an Earth-like atmosphere. Figure 8 shows the general effect of changing the optical depth on surface temperatures, for a constant $T_{eff} = 255K$.

Figures 9, 10, and 11 show how an Earth-like planet is affected by changing the albedo, the distance from the planet to the star, and the temperature of the star respectively, for a constant $\tau = 0.836$. Further discussion of this is given in Section 3.1.

2.9. *Temperature Distribution*

It should be noted that $T_{surface}$ is not necessarily a uniformly distributed temperature, but instead varies depending upon atmospheric circulation patterns. In general,

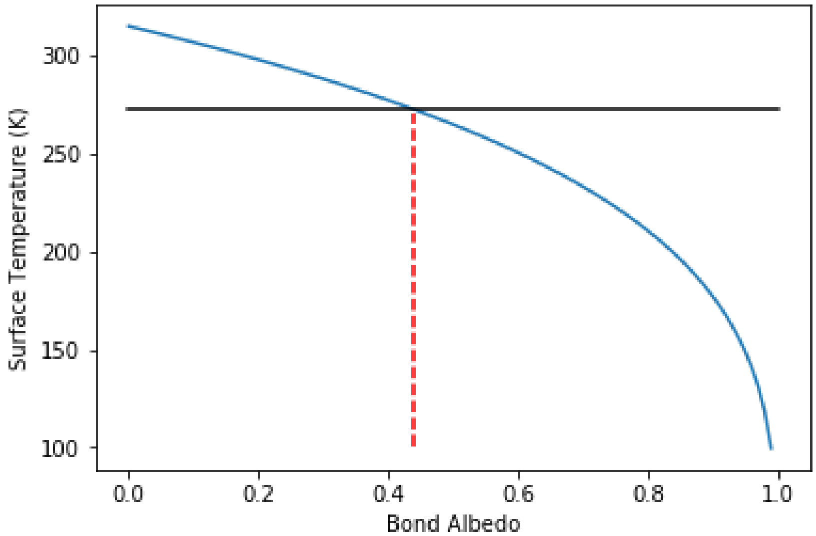

Figure 9. A graph showing the relationship for the surface temperature of the planet with a change in albedo. All other conditions, including atmospheric composition and mass, are identical to that of the Earth.

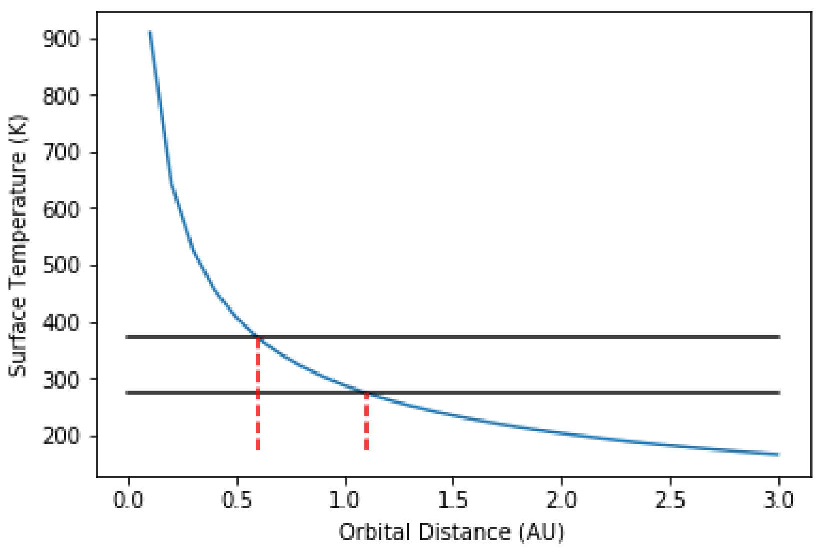

Figure 10. A graph showing the relationship for the surface temperature of the planet with a change in orbital distance. All other conditions, including atmospheric composition and mass, are identical to that of the Earth.

$T_{surface}$ will vary as a function of the longitude of the planet due to the change in stellar flux, as shown in figure 12.

In addition, the profile of $T_{surface}$ will depend on the latitudinal distribution of energy. For a planet with a sufficiently large difference between its rotational period and its orbital period, the distribution can be modelled as roughly uniform (as was assumed in the derivation of Equation 1).

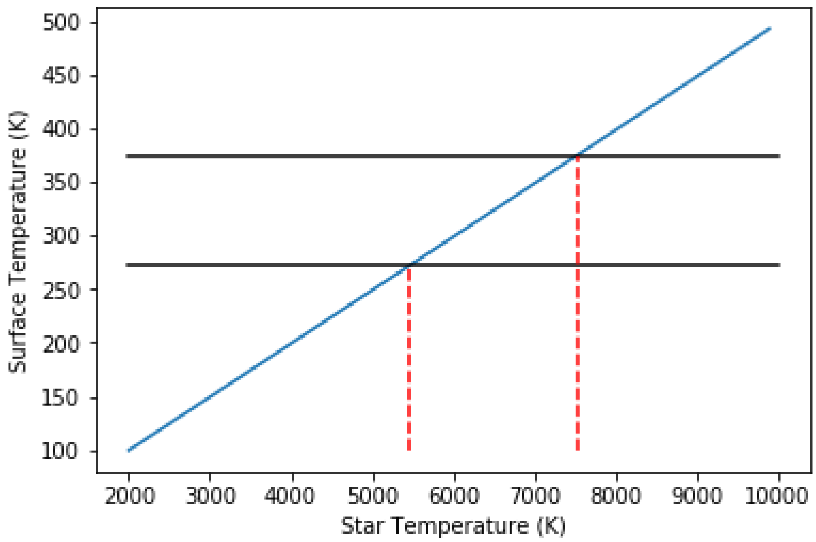

Figure 11. A graph showing the relationship for the surface temperature of the planet with a change in T_{star}. All other conditions, including atmospheric composition and mass, are identical to that of the Earth.

However, this does not necessarily hold in planets that exhibit tidal locking. Tidal locking is the phenomenon where the rotation rate and the orbital period of a planet result in one side of the planet always facing its star (or one side of a moon facing its planet). This in turn can result in a high temperature gradient where one side is far hotter than the other side. One example of tidal locking includes the Moon and the Earth. In addition, several of the exoplanets orbiting Gliese 581 are believed to be tidally locked, including Gliese 581d (Makarov et al., 2012). However, the presence of a weak Coriolis force on tide-locked habitable-zone planets can lead to reasonably horizontally uniform atmospheric temperatures (Joshi et al., 1997; Merlis and Schneider, 2010). As a result, incorporating tidally-locked planets into this model requires further development.

3. IMPLICATIONS

3.1. *The Habitability Question*

One immediate application of exoplanet temperature models is that it becomes possible to make estimates for the boundaries of the habitable zones, and thus determine whether a planet can sustain life. A typical definition of the habitable zone uses the assumption that life can only develop on planets where liquid water is in abundance, so that the habitable zone is the region in which the temperature range is bounded between 273K and 373K. (Kasting et al, 1993). For the purposes of this paper, the habitable zone shall be considered as the range of conditions which result in a planet having a surface temperature of between 273K and 373K. Unless otherwise specified, all constants will be set at Earth values.

As shown in Figure 8, if all other factors are kept constant, then the acceptable optical depth range for the Earth is between $\tau = 1.0849$ and $\tau = 5.4373$. This places

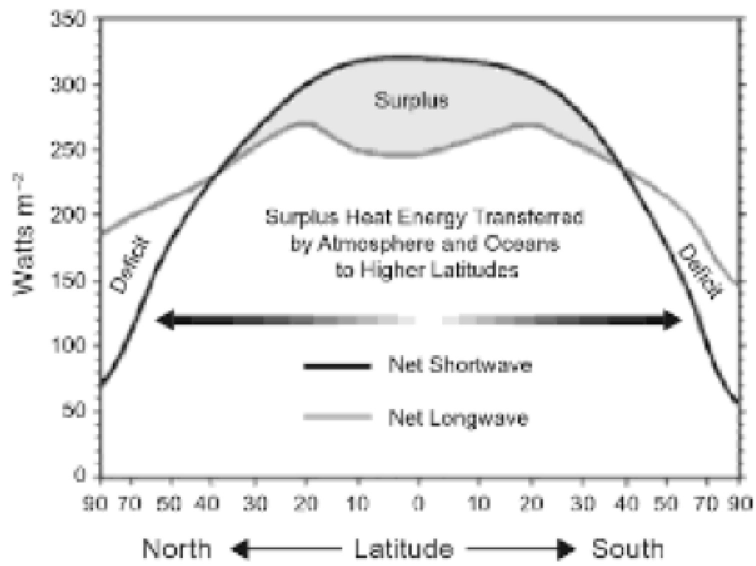

Figure 12. A graphical representation of the energy surplus and deficit generated at the poles and equator respectively for the Earth. Taken from Showman et al, 2010.

limits on the atmospheric composition of the Earth for it to remain habitable. In addition, as shown in Figures 9, 10, and 11, various limits for the albedo, orbital distance, and star temperature exist for the habitable zone for the Sun and Earth, given that all other factors are identical to that of the Earth. The maximum albedo in order for the Earth to remain in the habitable zone is 0.421. An albedo of 0 (i.e: maximal absorption) would still be within the habitable zone. The limits for orbital distance of the Earth from the Sun are 0.557AU and 1.156AU. The limits for the Sun's temperature are 5511K and 7562K.

It is also of interest to analyse how the habitable zone is affected by atmospheric composition. Considering a planet for which T_{eff}, ρ_o, and M_E is equal to that of the Earth, Figure 6 shows us that all mixtures of N_2 and O_2 result in a temperature that is outside the habitable zone. On the other hand, Figure 5 indicates that it is possible for an $H_2 - N_2$ atmosphere to be in the habitable zone, provided that the H_2 percentage is greater than 66.4%. Figure 7 shows that a very small percentage of CO_2 can result in a habitable planet with a $CO_2 - N_2$ atmosphere, but the percentage of CO_2 cannot exceed 10.3%. Figure 4 also shows that atmosphere surface density is significant. For example, for a 100% O_2 atmosphere, a temperature of 273K can be reached if $\rho_o = 1.5kg\ m^{-3}$, whilst if the Earth's atmosphere were to exceed $7.3kg\ m^{-3}$, the temperature would exceed 373K.

3.2. *Limitations of Model*

It is important to note that there are several limitations to the above model. The model in this paper for calculating surface temperature fails to consider any other sources of heat for a planet, such as large scale exothermic chemical processes due to geological activity. As a result, it may underestimate the temperature of certain exoplanets.

One example of a neglected factor is radioactive decay, caused in Earth-like planets by isotopes including ^{146}U, ^{40}K, and ^{142}Th. On Earth, heating from radioactive decay produces a heat flux of $0.08 Wm^{-2}$ (Davies, 1999). In addition, due to the model ignoring radioactive decay, the model does not apply to free-floating planets. Free-floating planets (or rouge planets) are objects of sufficient mass that do not orbit a star. It has been theorised that a free-floating planet that is optically thick could sustain a reasonable temperature through radioactive decay (Stevenson, 1999). For example, if Titan were a rogue planet with no incident stellar flux, Titan would need 1.6 Wm^{-2} of geothermal heat to maintain its current surface temperature, or an atmospheric opacity of 20 times its present amount with 0.1 Wm^{-2} of geothermal heat. (Gilliam and McKay, 2011). In addition, a rogue planet of Earth-like composition and age could maintain a subglacial liquid ocean (a so-called 'Steppenwolf planet') if it were approximately 3.5 times more massive than Earth, corresponding to around 8 km of ice (Abbot and Switzer, 2011). This provides a possibility for a planet to sustain a temperature between 273-373K, while existing outside of the conventional habitable zone distances from stars. By failing to include this class of planets, it has potentially placed an unnecessary restriction on the criteria for a planet to be habitable. As a result, further work must be done in this area.

In addition, the impact of tidal heating will need to be incorporated into any future work, as this can be a significant factor in the heating of exoplanets. For example, Jupiter's moon Io has a significant heating flux ($h = 2Wm^{-2}$) from tidal heating (McEwen et al., 2004; Barnes et al., 2009). If the tidal heating of an exoplanet is close to that of Io then the development of life is unlikely given the surface conditions. (Barnes et al., 2009). However, if the tidal heating rate is less than the minimum required to initiate plate tectonics, then it is possible that CO_2 will not be recycled by subduction. This could result in a runaway greenhouse effect where a positive feedback causes CO_2 to build up in the atmosphere, potentially leading to temperatures on the scale of those seen in Figure 7. (Barnes et al. 2009).

Furthermore, this model ignores the effect of clouds and aerosols. The modelling of clouds in GCMs is significant because an increase in cloud cover will increase the Bond albedo of the planet, and will alter the optical depth of the atmosphere. The impact of clouds on surface temperature depends on a number of variables, including stellar flux. For example, for M-type stars, CO_2 clouds lead to 6K of additional warming at best, but for F-type stars, they can result in up to 30K of additional warming. (Kitzmann, 2017). Cloud cover may also be a significant factor in Earth-like planets that contain H_2O in the atmosphere, as well as planets that have an ocean.

Aerosols are compounds that can act as the opposite of a GHG by resulting in a cooling of the planet in question, with a major example being SO_2. By ignoring the effect of aerosols, it is possible that overestimates of surface temperatures may be calculated when modelling certain exoplanets with a high proportion of SO_2 or other aerosols. Certain bodies, such as Titan, also exhibit a significant anti-greenhouse effect, an phenomenon where the atmosphere has a lower transmittance for incoming radiation than outgoing radiation, resulting in a cooling effect. In the case of Titan, this is due to the formation of organic molecules in the upper atmosphere, causing the atmosphere to be opaque at ultraviolet and visible wavelengths, but transparent to IR radiation (McKay et al., 1999).

4. CONCLUSIONS

In conclusion, there are a number of factors that can influence the surface temperature of an exoplanet, with major factors being the properties of the stellar body it orbits (star temperature and radius), the distance between the stellar body and the planet, the atmospheric composition, and various other properties of the planet including its mass and atmospheric surface density.

By modelling a planet with identical properties to that of the Earth, various atmospheric compositions and their effect on surface temperatures were investigated. Various graphs were plotted to demonstrate the relationships between variables, such as surface temperature and planetary mass, as well as surface temperature and atmospheric surface density. From this, several limits on the habitable zone were found.

Considerable work would need to be done to build a more accurate, and general model for surface temperatures. This includes the potential introduction of an anti-greenhouse effect for the consideration of aerosols for which McKay et al, 1999 offers a potential example of how this could be approached. Additional factors that should be considered are the impact of cloud cover on optical depth and albedo, and the heating effect due to radioactive activity and tidal heating.

I would like to thank all of the people who have helped in the development of this paper. This includes my advisor Adam Burrows who has provided excellent feedback throughout the process of formulating and writing this paper.

This paper represents my own work in accordance with University regulations.
- Sam Moore

REFERENCES

Butler, R.P. et al. 2004, ApJ, 617, 1

Demory, B. et al. 2011, ApJ, 735, 1

Kaltenegger, L. 2017, ARA&A, 55, 433-485

Palle, E. et al. 2016, GRL, 43, 9

Coddington, O. 2016, AMS, 97, 7

Basilevsky, A.T., Head, J.T. 2003, RPP, 66, 10

Domingue, D.L. et al. 2007, SSRv, 131, 1-4

Kochanov et al. 2016, ApJ, 177, 15-20

Pierrehumbert and Gaidos 2011, ApJ, 734, 1

Badescu, V. 2009, CEJP, 8, 3

Kasting et al. 1993, Icarus, 100, 1

Gillian and Mckay 2011, PSS, 59, 9

Abbot and Switzer 2011, APJ, 735, 2

Makarov et al. 2012, APJ, 761, 2

Joshi et al. 1997, Icarus, 129, 2

Merlis and Schneider 2017, JAMES, 2, 4

Stevenson 1990, Nature, 400, 32

Barnes et al. 2009, APJ, 700, 1

Kitzmann 2017, AA, 600, A111

McKay et al. 1999, Icarus, 137, 1

Showman, Adam P., James YK Cho, and Kristen Menou. “Atmospheric circulation of exoplanets.” Exoplanets 526 (2010): 471-516.

McEwen, A. S., Keszthelyi, L. P., Lopes, R., Schenk, P. M., and Spencer, J. R. 2004, Jupiter: The planet, Satellites and Magnetosphere, ed. F. Bagenal, T. E. Dowling, and W. B. McKinnon (Cambridge: Cambridge Univ. Press), 307

Davies, G. ”Dynamic Earth” (Cambridge: Cambridge University Press), p. xi 458 (1999)

Williams, D. R. (2004). Earth Fact Sheet. Retrieved from https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html [Accessed: 27/12/2017]